

# Multiplicities for non-extensive statistical model at LHC energies

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## Abstract

We analysed the identified hadron multiplicities predictions of the generalised thermodynamical model of the multiparticle production processes with non-extensive statistic. The multiplicities measured recently at LHC experiments are found to be consistent with this approach and thermodynamical parameter values found already analysing transverse momentum distributions. The information about the mechanism of the strangeness suppression has been derived to some extend from multistrange hadron abundances.

## INTRODUCTION

The thermodynamical picture of particle creation in hadronic collisions was the first and quite successful attempt to describe it. The elaborated and complete theory of this kind was presented in serial of papers by Hagedorn [see [1] and references therein]. The idea of the fireball with the proposition that “all fireballs are equal” give very strong predictions concerning produced particle spectra.

One of the predictions was that the temperature of the “hadronic soup” (precisely defined) could not exceed a universal constant  $T_0$  of order of 160 MeV. This value comes from examination of elementary particle mass spectrum not as a result of the procedure of parameter adjusting using multiparticle production (e.g., transverse momenta) data. The Hagedorn theory were abundant (for some time) when more sophisticated, jet or QCD based ideas appeared [2]. One of the reasons was the failure of the high-transverse momenta description. The temperature of the fireball is defined as the parameter in the classical Boltzmann exponential term of the probability weights for phase space average occupation numbers. This defines the (asymptotic) form of the distribution of transverse momentum of particles created from decaying fireballs. It was found that at high and very high interaction energies the predicted exponential fall do not agree with the observed high  $p_\perp$  behaviour.

Successes of QCD based description of the hard processes gave deep insight into the nature of physics involved, and common believe that this is just the right theory of strong interactions, makes the thermodynamical approach only one, very approximate, simple and naive, tool of limited applicability and thus limited significance. But on the other hand, the simplicity of the theory and notorious constant lack of the effective QCD theory of soft hadronization processes give a hope that the fireball idea can be enriched, modified and can become important again.

The identified hadron ratios have been measured with all LHC detectors and results were compared with high-energy event generators available in the market. The comparison was not very satisfactory.

The ATLAS results on strange particle production are compared with various Monte Carlo simulation models [3] and again some substantial disagreements are found in distributions of transverse momentum.

The LHCb prompt hadron measurement ratios [4] were compared with several Monte

Carlo generator results and it was concluded that none was able to describe well all observables, e.g., significant underestimation of  $(K^+ + K^-)/(\pi^+ + \pi^-)$  has been observed with the Perugia 0 and Perugia NOCR generators.

The ALICE data on  $K^*$  and  $\psi$  production [5] have been compared to a number of PYTHIA [6] tunes and the PHOJET[7] event generator. None of them gives a fully satisfactory description of the data.

ALICE Collaboration measured also neutral pion and  $\eta$  mesons [8] and concluded that calculations cannot provide a consistent description at 7 TeV, as well multi-strange baryons  $\Xi$ s and  $\Omega$ s [9], and found that particle yields,  $\langle p_\perp \rangle$ , and the spectra in the intermediate  $p_\perp$  range are not well described by the PYTHIA Perugia 2011 tune Monte Carlo event generator specially for  $\Omega$ .

## PARTITION FUNCTION

The Hagedorn idea was used again to describe the identified particle multiplicities in hadronization both, in  $e^+e^-$  annihilation and hadronic collisions. The *grand canonical* formalism of Hagedorn was replaced in the serial of papers by Becattini and co-workers [10] by the *canonical* one, much relevant for studies of small systems like primary created fireballs for which the requirement of exact conservation of some quantum numbers seems important. In general, thermodynamics of the system is determined by the partition function which can be written as

$$\mathcal{Z}(Q^0) = \sum_Q \delta(Q - Q^0) \prod_{i,j} x_{jk}^{\nu_{jk}}, \quad (1)$$

where  $x$  is the Boltzmann exponential factor  $x = \exp(-E_{jk}/T)$ ,  $j$  and  $k$  enumerate particle types and momentum cells,  $Q^0$  is the initial fireball quantum number vector and  $Q$  is the respective vector of the particular state, and  $\nu_{jk}$  is the occupation number. For bosons one has  $\nu_{jk} = 0, 1, 2, 3 \dots$  and  $\sum_{\nu_{jk}} x_{jk}^{\nu_{jk}} = 1/(1 - x_{jk})$ , while for fermions  $\nu_{jk} = 0, 1$  and  $\sum_{\nu_{jk}} x_{jk}^{\nu_{jk}} = 1 + x_{jk}$ . Thus introducing Fourier transform of  $\delta$  (and reducing vector  $Q$  to

3-dimensional: charge, baryon number and strangeness) Eq.(1) becomes

$$\begin{aligned} \mathcal{Z}(Q^0) = & \frac{1}{(2\pi)^3} \int d^3\phi e^{iQ^0\phi} \times \\ & \times \exp \left\{ \sum_{j=1}^{n_B} \sum_k \log(1 - x_{jk} e^{-iq_j\phi})^{-1} + \right. \\ & \left. + \sum_{j=1}^{n_F} \sum_k \log(1 + x_{jk} e^{-iq_j\phi}) \right\}, \end{aligned} \quad (2)$$

where  $q_j$  is the quantum number vector of the particle  $j$ .

The next step is to turn the summation over phase space cells to an integration over momentum space

$$\sum_k \longrightarrow w_j \frac{V}{(2\pi)^3} \int d^3p \quad (3)$$

where  $w_j$  is the weight factor associated with the particle  $j$ . The first guess is that it should be equal to  $(2J_j+1)$  and counts spin states. However, this does not to be so simple (see, e.g.,[6]) and other solutions introducing factors responsible for some wave-function normalisation, which should disfavour heavier states were found to be preferable by measurements. We will discuss this point later on.

With the Eq.(2) we are ready for detailed numerical calculations.

### Average multiplicities - Boltzmann statistic

With the known partition function  $\mathcal{Z}$  the average characteristics of the system can be obtained in an usual way. The average occupation number  $\nu_j$  representing the mean multiplicity of the particles of the kind  $j$  is (introducing the fictious fugacity  $\lambda$ ) given by

$$\langle n_j \rangle = \left. \frac{\partial}{\partial \lambda} \log(\mathcal{Z}(\mathbf{Q}^0, \lambda)) \right|_{\lambda=1}, \quad (4)$$

thus

$$\langle n_j \rangle = w_j \frac{V}{(2\pi)^3} \frac{1}{(2\pi)^3} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} d^3\phi \int d^3p [e^{E/T} e^{i\mathbf{q}_j\phi} \pm 1]^{-1}, \quad (5)$$

where the upper sign is for fermions and the lower is for bosons. Because the  $e^{-E/T}$  factor is expected to be small for all particles except pions ( $T \approx 100$  MeV) the following

approximation can be made in such cases

$$\frac{1}{e^{E/T} e^{i \mathbf{q}_j \phi} \pm 1} \longrightarrow e^{-E/T - i \mathbf{q}_j \phi} \quad (6)$$

and then

$$\begin{aligned} \langle n_j \rangle &\approx \frac{1}{\mathcal{Z}(\mathbf{Q}^0)} \frac{1}{(2\pi)^3} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} d^3\phi \mathcal{Z}(\mathbf{Q}^0) e^{-i \mathbf{q}_j \phi} w_j \frac{V}{(2\pi)^3} \int d^3p e^{-E/T} = \\ &= \frac{\mathcal{Z}(\mathbf{Q}^0 - \mathbf{q}_j)}{\mathcal{Z}(\mathbf{Q}^0)} w_j \frac{V}{(2\pi)^3} \int d^3p e^{-E/T} . \end{aligned} \quad (7)$$

The  $\mathcal{Z}(\mathbf{Q}^0 - \mathbf{q}_j)/\mathcal{Z}(\mathbf{Q}^0)$  term in Eq.(7) represents classical chemical potential

### Modification of the statistical hadronization model

The conventional Boltzmann-Gibbs description shown above was modified to allow the description of the systems of not-completely-free particles: the correlation “strength”, however defined, was introduced with the help of the new *non-extensivity* parameter and the new statistics, which in such case has to be also *non-extensive*. In the limit of the absence of correlations the new description approach the Boltzmann form.

There could be infinitely many “generalised” statistics which fulfil such requirements. Following ‘Ockham razor’ we should choose the one which is simple and has well defined theoretical background. In this present paper we test the possibility, proposed by Tsallis [11], based on the modification of the classical entropy definition

$$S_{\text{BG}} = -k \sum_i^W p_i \ln p_i \quad (8)$$

by the new one

$$S_q = k \frac{1}{q-1} \left( 1 - \sum_i^W p_i^q \right) \quad (9)$$

with the new parameter  $q$  called the non-extensivity parameter. This modification has been adopted in other physical applications (see, e.g., [12]).

Maximisation of the entropy requirement with the total energy constraint

$$\frac{\sum_i p_i^q E_i}{\sum_i p_i^q} = E_0 \quad (10)$$

leads to the probability of realisation of the state  $i$  (with energy  $E_i$ ) given by

$$p_i^q = \frac{1}{\mathcal{Z}_q} [1 - (1 - q)/T_q(E_i - E_0)]^{q/(1-q)} , \quad (11)$$

where  $\mathcal{Z}_q$  is the normalisation constant related to  $\mathcal{Z}(\mathbf{Q}^0)$  of Eq.(1) where the Boltzmann terms  $x$  could be replaced by the probabilities of the form given in Eq.(11).

Equation (11) can be rewritten introducing new symbol,  $e_q$ , defined as

$$(e_q)^x = [1 + (1 - q)x]^{q/(1-q)} \quad (12)$$

(for completeness, we should mention that in the  $q=1$  limit we have, as we should,  $e_1^x = e^x$ ).

Then

$$p_i^q \sim [1 - (1 - q)/T_q(E_i - E_0)]^{q/(1-q)} \sim e_q^{-E_i/T_q} \quad (13)$$

and the modified partition function can be written in the form

$$\mathcal{Z}_q(\mathbf{Q}) = \sum_{\text{states}} \frac{w_j}{(2\pi)^3} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} d^3\phi e_q^{-E/T} e^{i(\mathbf{Q}_0 - \mathbf{Q}) \cdot \phi} . \quad (14)$$

## RESULTS

We have evaluated  $\mathcal{Z}_q$  functions (and  $\mathcal{Z} = \mathcal{Z}_1$ ) for a variety of thermodynamical parameter values  $T$ ,  $V$ , and strangeness for  $\mathbf{Q}$  values which cover the production of over 100 hadrons of masses below 2 GeV/c<sup>2</sup>. All decays of short-lived particles were then performed. In the Ref. [13] we have shown the effect of exchange the statistical factor from the Boltzmann one to the Tsallis modified exponential factor on the  $p_\perp$  distribution. We have found that the description of transverse momenta with non-extensive statistics works quite well and follow the earlier found [14] trends.

The measurements of some identified particle ratios done mostly by the ALICE Collaboration gives the opportunity to test the statistical model of particle flavours creation in the new, higher energy region. The lower energy jet hadronisation results have been analysed by the serial of papers by Becattini and others ([15, 16]). It has been shown that the micro-canonical Boltzmann description works well for  $e^+e^-$  from  $\sqrt{s} \approx 10$  [17] to 91 GeV at Ref. [15] and  $pp$  and  $p\bar{p}$  interactions up to SPS energies,  $\sqrt{s} \approx 900$  GeV [16].

The comparison of the results obtained with Boltzmann (non-extensivity parameter  $q = 1$ ) and Tsallis (with the value of  $q = 1.12$  as expected from Ref. [13]) is shown in Fig. 1. We can see the clear enhancement of the exponential 'tail'.

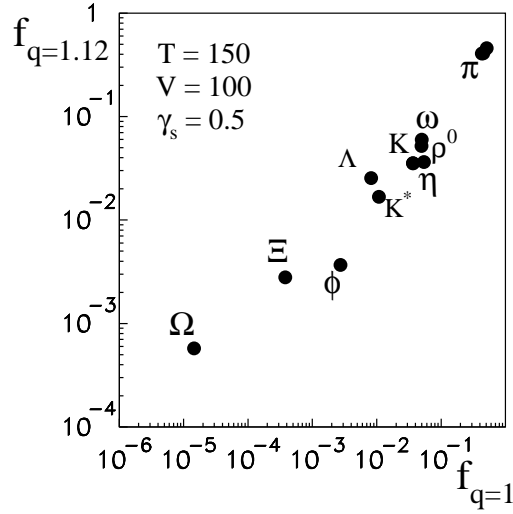


FIG. 1: Relative particle multiplicities,  $f = (n_i/n_{ch})$ , obtained for the Tsallis statistics (with  $T = 150$ ,  $V = 100$  and  $q = 1.12$ ) compared with the same ratios for the Boltzmann statistics (with  $T = 150$ ,  $V = 100$  and  $q = 1$ ).

The effect of generalisation of the statistics is seen also in the Fig 2. The strong effect observed when we compare Boltzmann  $q$  parameter 1.0 results with the higher  $q$  values is clear. The analysis of LHC data on transverse momenta suggests that the appropriate value at 7 TeV centre of mass energy is about equal to 1.12. For small changes of  $q$  the differences are still not very significant.

We have checked different particle multiplicities dependencies on main thermodynamical hadronization parameters  $T$  and  $V$ . Some results are shown, as an example, in Fig.3

The effect of volume  $V$  have to be taken with care, because the spacial and temporal history of the hadronisation process is not exactly known. It is expected that the canonical picture should take into account the multichain idea (e.g., [10, 18]) of decomposition of the 'hadronic soup' to chain of independently hadronized objects/fireballs. For the Boltzmann statistics, by the definition of extensivity, the sum of many hadron sources is equivalent to one big source [17]. This is, in general, not the case of Tsallis non-extensive statistics. But we can say that the strength of the non-extensivity is still rather small and the effect of subdivision of the hadronisation volume does not change the conclusion about the identified particles ratios. Another point is related to the effect of the canonical treatment of small fireballs which relates to the suppression of strange quark (and diquark, or strange diquark)

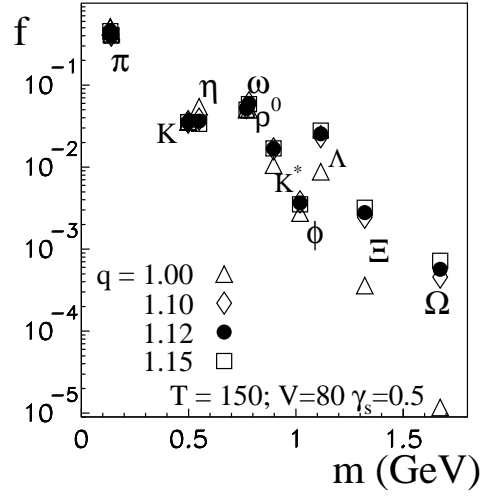


FIG. 2: Relative particle multiplicities,  $f = (n_i/n_{ch})$ , obtained for Boltzmann and different Tsallis generalised statistics (with  $T = 150$ ,  $V = 100$ ).

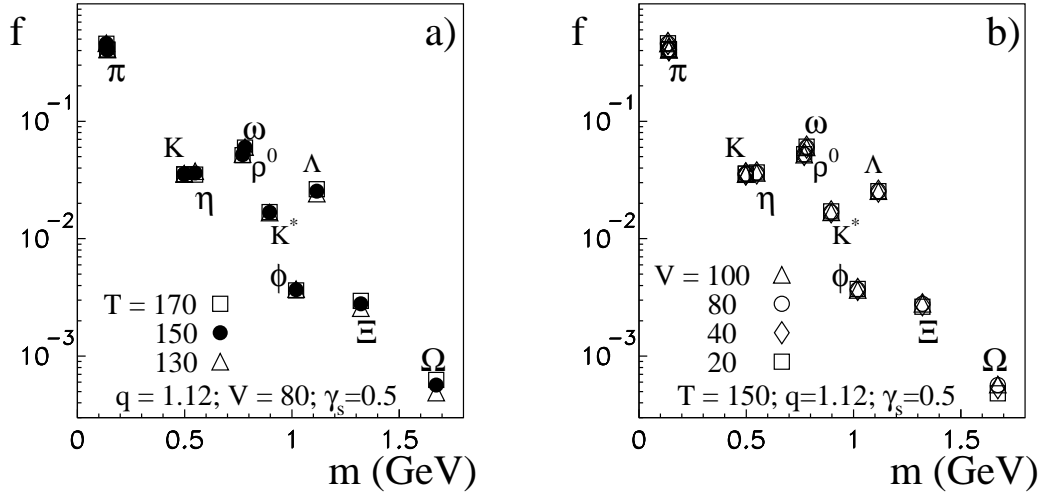


FIG. 3: Relative multiplicities,  $f = (n_i/n_{ch})$ , calculated for Tsallis statistics for different temperatures  $T$ , (left) and different hadronization volumes  $V$  (right).

what was mentioned already in [1]. We will come back to these additional suppressions later on.

Fig. 3 suggest that ratios of identified particle multiplicities are not sensitives to the thermodynamical parameters of the model of fireballs hadronization. It seems that the predictions can not be adjusted to the data, at least in the range of possible, reasonable



changes, which is quite small. They are as came out from integrations in Eq.(7) and modified statistics Eq.(14) with  $q$  fixed by  $p_\perp$  distribution fit. The comparison with the data could be thus the *experimentum crucis* of the thermodynamical models in general. Results for particle ratios are shown in comparison with data measured by the ALICE Collaboration in Tab.I. Thermodynamical parameter values were taken to reproduce roughly the data, where it is possible.

TABLE I: Ratios of particle multiplicities calculated for Boltzmann and Tsallis statistics with spin counting states weight factor equal to  $(2J + 1)$  and strangeness suppression  $\gamma_s = 0.5$  acting of strange quark contents compared with the measurement results from ALICE.

particle ratio	ALICE measurement	<i>Boltzmann</i>	<i>Tsallis</i>	<i>Tsallis</i>
		$V=50$ $T=160$ $q=1.00$	$V=80$ $T=170$ $q=1.12$	$V=80$ $T=150$ $q=1.15$
$\rho/\omega$	$1.15 \pm 0.2 \pm 0.12^a$	0.985	0.855	0.848
$\phi/(\rho + \omega)$	$0.084 \pm 0.013 \pm 0.012^a$	0.042	0.035	0.033
$K^{*0}/K^-$	$0.35 \pm 0.001 \pm 0.04^b$	0.337	0.466	0.466
$\phi/K^{*0}$	$0.33 \pm 0.004 \pm 0.05^b$	0.268	0.215	0.207
$\phi/\pi^-$	$0.014 \pm 0.0002 \pm 0.002^b$	0.0063	0.0080	0.0077
$\phi/K^-$	$0.11 \pm 0.001 \pm 0.02^b$	0.090	0.100	0.097
$\omega/\pi^0$	$0.6 \pm 0.1^c$	1.36	0.861	0.704
$\Omega/\Xi$	$0.067 \pm 0.01^d$	0.068	0.237	0.240
$\Omega/\phi$	$0.04 \pm .008^e$	0.119	0.362	0.403
$\eta/\pi^0$	$0.1067 \pm 0.0259 \pm 0.0212^f$	0.206	0.092	0.081

<sup>a</sup>1 GeV/c <  $p_\perp$  < 5 GeV/c [19]

<sup>b</sup>full phase space [5]

<sup>c</sup> $p_\perp > 2.5$  GeV/c [20]

<sup>d</sup> $m_\perp - m_0 > 0.3$  GeV [9]

<sup>e</sup> $p_\perp > 1$  GeV [5]

<sup>f</sup> $p_\perp > 0.55$  GeV/c [8]

We clearly see problems when comparing the ALICE results with any model predictions, whatever parameter set and model is taken. It could be the substantial and general disagreement (at least in some cases), however, we have the weight factor  $w_j$  in Eqs. (5,7) which give

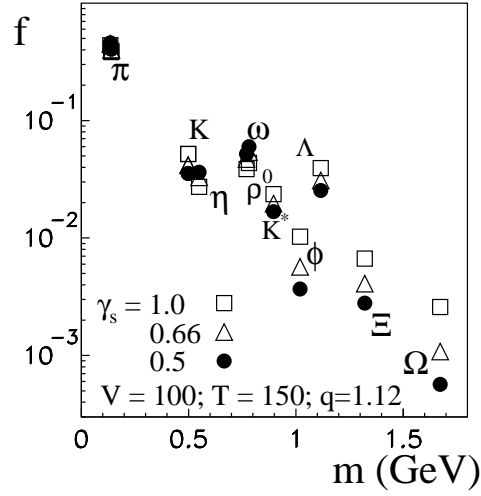


FIG. 4: Relative particle multiplicities,  $f = (n_i/n_{ch})$ , obtained for the Tsallis statistics and different values of the strangeness suppression parameter  $\gamma_s$  parameter ( $T = 150$ ,  $V = 100$ ,  $q = 1.12$ ).

us some hope and freedom to fit the model predictions. The expected form of  $(2J + 1)$  is, in general, modified since there had been experimentally found the suppression of  $K$ s with respect to non-strange particles. The factor  $\gamma_s$  was introduced and adjusted for obtain the correct value of  $K/\pi$  ratio. The general statement is that the strange phase space is not full available for particle production what can be realised by multiplying the partition function by given factor for each strange valence quark in the particle in question. The effect is of course significant for heavy, strange mesons, but not only. It is shown in Fig. 4

The strangeness suppression factor is one of the basic parameters in the jet fragmentation model introduced by Feynman and developed finally by the Lund group [6]. In the Lund jet fragmentation process new hadrons appear as a breaking of the colour field string stretched between quarks moving apart by the production of a new pair of quarks (sometimes di-quarks). If there is enough energy left, further breaks may occur and eventually only on-mass-shell hadrons remains. The creation of new quark-antiquark pair in the Lund model is a kind of the quantum tunnelling process, so it is expected that the heavy quark creation is suppressed. It is usually assumed that  $u : d : s \sim 1 : 1 : 0.3$ .

Additional  $w_j$  weight factor is related with the spin stated of newly created hadrons: for mesons pseudoscalar and vector states. The eventual suppression here is not defined in the Lund fragmentation model. The counting the spin states gives  $1 : 3$  ratio, but in the

JETSET model it is modification according to the 'tunnelling normalisation' and this ratio is eventually close to 1 : 1.

The situation with baryon creation in the Lund model is much more complicated. The tunnelling mechanism is also adopted here. We have here the probability of string breakup via diquark mode and further combination of quark and diquark. If we take into account the pop-corn mechanism of diquark breakups, and lack of general rules, we can have the number of parameters to be adjusted to the data comparable with the number of measured ratios to be used for the adjustment. The number of parameters describing the production of baryons measured with good accuracy in the experiments at LHC is at the moment higher than the number of such baryons itself [6].

The Lund model and particular JETSET hadronization generator is used also by the PHOJET [7] program package for the recent theoretical examination of the LHC data description and comparison. Some parameters in PHOJET are different than the default Lund model values.

We first discuss the possibility of introducing the strangeness suppression factor  $\gamma_s$  for

$$w_j = (2J + 1) \times (\gamma_s)^{N_j} \quad . \quad (15)$$

The  $N_j$  is the 'degree of strangeness' which is in fact not exactly defined. It should be related to the contents of the particle  $j$ . Three possibilities are rather natural.

$$N_j = \begin{cases} S & \text{strangeness of the particle } j \\ n_s & \text{number of strange (or antistrange) valence quarks of } j \\ n_{s\bar{s}} & \text{number of } s\bar{s} \text{ pairs involved to create the particle } j \end{cases} \quad (16)$$

The difference could be seen in the comparison of  $K$  and  $\phi$  weights. For direct  $K$  they are  $\gamma_s$ ,  $\gamma_s$ , and  $\gamma_s$ , while for direct  $\phi$  they are 1,  $\gamma_s^2$ , and  $\gamma_s$ , for the first, the second and the third possibility, respectively. The actual situation is more complicated, because of the effects of decays heavy resonances, so to see the effect complete calculations have to be performed.

Some example results are given in the Tab. II.

It can be seen that calculated ratios shown in Tab. II that the case ( $N_j = n_s$ ) works well for strange mesons and the simple  $(2J + 1)$  factor results not far from measurements.

TABLE II: Ratios of particle multiplicities calculated for  $T = 150$ ,  $V = 80$  and  $q = 1.12$  with different strangeness suppression factor definition with the strangeness suppression factor equal to  $\gamma_s = 0.5$  compared with the measurement results.

particle ratio	ALICE measured ratios	$n_s$	$n_{s\bar{s}}$	$S$
$\rho/\omega$	$1.1 \pm 0.2 \pm 0.12^a$	0.862	0.870	0.888
$\phi/(\rho + \omega)$	$0.084 \pm 0.013 \pm 0.012^a$	0.0360	0.0718	0.142
$K^{*0}/K^-$	$0.35 \pm 0.001 \pm 0.04^b$	0.462	0.423	0.382
$\phi/K^{*0}$	$0.33 \pm 0.004 \pm 0.05^b$	0.219	0.440	0.751
$\phi/\pi^-$	$0.014 \pm 0.0002 \pm 0.002^b$	0.0080	0.0160	0.0301
$\phi/K^-$	$0.11 \pm 0.001 \pm 0.02^b$	0.101	0.186	0.287
$\omega/\pi^0$	$0.6 \pm 0.1^c$	0.894	0.893	0.870
$\Omega/\Xi$	$0.067 \pm 0.01^d$	0.233	0.219	0.233
$\Omega/\phi$	$0.04 \pm .008^e$	0.329	0.135	0.082
$\eta/\pi^0$	$0.1067 \pm 0.0259 \pm 0.0212^f$	0.100	0.102	0.103

Further 'tuning' can involve the value of the strangeness suppression factor  $\gamma_s$ . we have checked tree values which have been used in the literature: 0.5 originally proposed by Feynman in the jet fragmentation model [2] and still of use [10], 2/3 used successfully by Becattini [21] for the  $e^+e^-$  data and  $p\bar{p}$  results from SPS, and  $\gamma_s = 1$  as the limit of no strangeness suppression. The results are shown in Tab.III

The general agreement, at least for mesons, is seen for  $\gamma_s \approx 0.5 - 0.66$ .

The discrepancy exists still in ratios involved baryons. As it has been said there is a great degree of freedom to modify baryon multiplicities. The diquark suppression factor  $\gamma_{qq}$  is the one possibility which we used and another factor was introduced specially for  $\Omega$  baryons  $\gamma_{ss}$  related to creation of double strange diquark.

In the Tab. IV we present results for the  $\gamma_{qq}$  and  $\gamma_{ss}$  equal to 0.5.

Because of relatively limited amount of data we do not wish at the moment to go further with 'tuning' the parameters ( $\gamma_s$ ,  $\gamma_{qq}$  and  $\gamma_{ss}$ ).

TABLE III: Ratios of particle multiplicities calculated for  $T = 150$ ,  $V = 80$  and  $q = 1.12$  with different strangeness suppression(for  $n_s$ ) factor values and spin counting states exactly equal to  $(2J + 1)$  compared with the measurement results.

particle ratio	ALICE results	$\gamma_s = 1$	$\gamma_s = 0.66$	$\gamma_s = 0.5$
$\rho/\omega$	$1.15 \pm 0.2 \pm 0.12^a$	0.882	0.867	0.861
$\phi/(\rho + \omega)$	$0.084 \pm 0.013 \pm 0.012^a$	0.137	0.0618	0.0360
$K^{*0}/K^-$	$0.35 \pm 0.001 \pm 0.04^b$	0.429	0.453	0.462
$\phi/K^{*0}$	$0.33 \pm 0.004 \pm 0.05^b$	0.433	0.289	0.219
$\phi/\pi^-$	$0.014 \pm 0.0002 \pm 0.002^b$	0.0234	0.0126	0.00802
$\phi/K^-$	$0.11 \pm 0.001 \pm 0.02^b$	0.186	0.131	0.101
$\omega/\pi^0$	$0.6 \pm 0.1^c$	0.787	0.857	0.894
$\Omega/\Xi$	$0.067 \pm 0.01^d$	0.443	0.303	0.233
$\Omega/\phi$	$0.04 \pm .008^e$	0.585	0.416	0.329
$\eta/\pi^0$	$0.1067 \pm 0.0259 \pm 0.0212^f$	0.084	0.094	0.161

The ALICE Collaboration published data showing the particle ratio as a function of transverse momentum of the particles. Taking into account that the modification of the statistics of the multiparticle production process was developed primary for the transverse momentum description this kind of data could be valuable to verify the model. Comparison of our final model prediction and the data are shown in Fig. 5.

The Boltzmann statistics results of the fit with parameters of Ref. [16] are also given for comparison. As it is seen, the standard statistics does not work very well for the LHC ALICE data shown in Fig. 5, while the modified, Tsallis statistics with our chosen suppression factors reproduces the data quite better.

## CONCLUSIONS

We have shown that the introduction of non-extensive statistics to the thermodynamical theory of multiparticle production in hadronic collisions can give a consistent description of identified particle multiplicity ratios. The thermodynamical model parameters found analysing the transverse momentum distributions measured at 7 TeV could be used without

TABLE IV: Ratios of particle multiplicities calculated for  $T = 150$ ,  $V = 80$  and  $q = 1.12$  with different strangeness suppression factor 0.5 and 0.66 (and  $\gamma_{qq} = \gamma_{ss} = 0.5$ ) compared with the measurement results.

particle ratio	ALICE results	$\gamma_s = 0.5$	$\gamma_s = 0.66$
$\rho/\omega$	$1.15 \pm 0.2 \pm 0.12^a$	0.862	0.867
$\phi/(\rho + \omega)$	$0.084 \pm 0.013 \pm 0.012^a$	0.036	0.0620
$K^{*0}/K^-$	$0.35 \pm 0.001 \pm 0.04^b$	0.462	0.453
$\phi/K^{*0}$	$0.33 \pm 0.004 \pm 0.05^b$	0.226	0.302
$\phi/\pi^-$	$0.014 \pm 0.0002 \pm 0.002^b$	0.0091	0.0147
$\phi/K^-$	$0.11 \pm 0.001 \pm 0.02^b$	0.104	0.136
$\omega/\pi^0$	$0.6 \pm 0.1^c$	0.907	0.872
$\Omega/\Xi$	$0.067 \pm 0.01^d$	0.0797	0.103
$\Omega/\phi$	$0.04 \pm .008^e$	0.0363	0.0447
$\eta/\pi^0$	$0.1067 \pm 0.0259 \pm 0.0212^f$	0.111	0.105

readjustment for identified particle multiplicity data description. The standard strangeness suppression factor  $\gamma$  of about 0.5-0.6 with additional suppression of diquark and strange diquark phase space works well for average as well as  $p_\perp$  dependence of particle ratios.

Proposed treatment of hadronization can put some new light on the particle creation physics and it is somehow complementary to the jet type non-perturbative QCD models.

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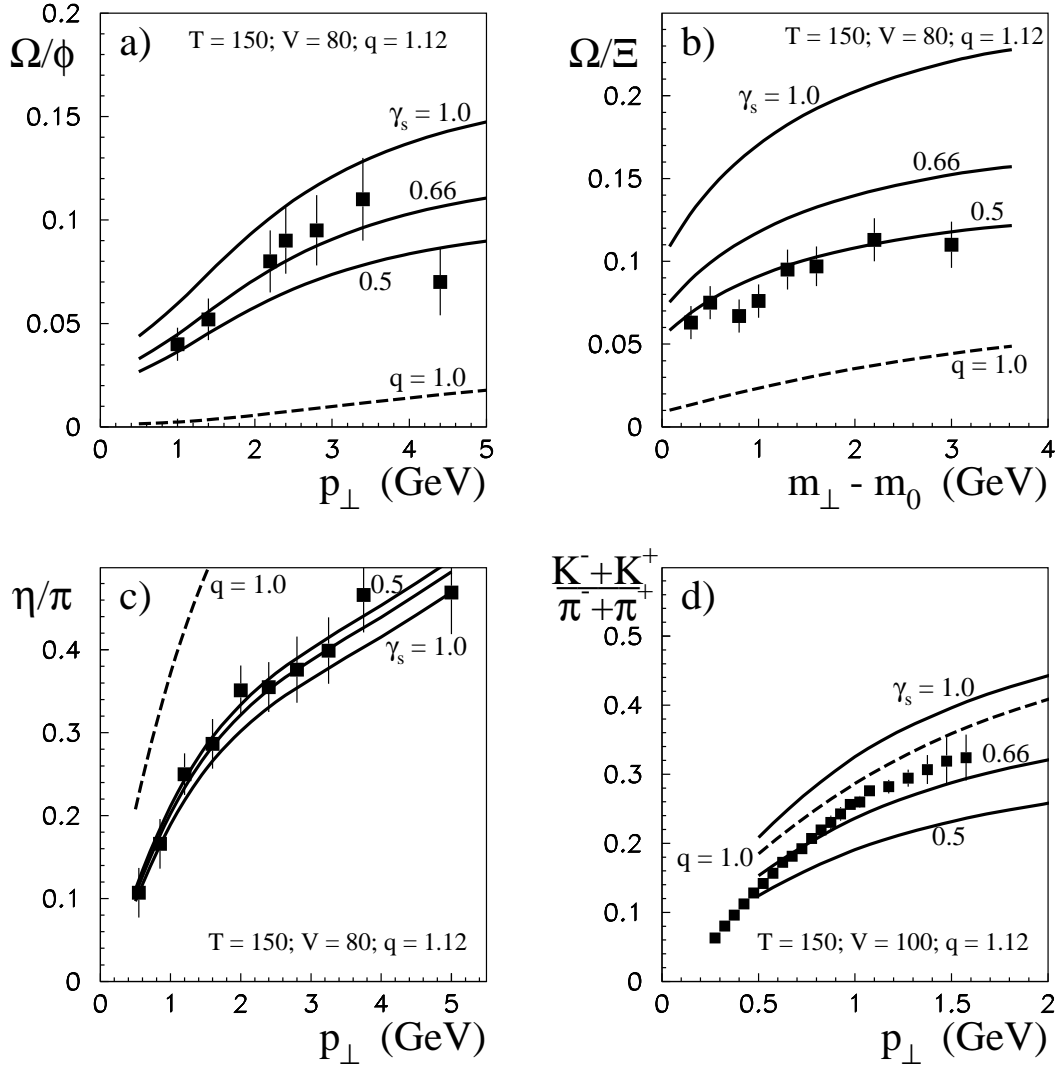


FIG. 5: Identified particles ratios as a function of transverse momentum (transverse mass for  $\Omega/\Xi$ ) for  $T = 150$ ,  $V = 80$ ,  $q = 1.12$  and strangeness suppression  $\gamma_s = 0, 5$  acting of strange quarks (or antiquarks) contents of the particle in comparison with ALICE data.

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